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Analysis the problem of thin plate based on cubic hermite interpolation

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Abstract. The wavelet finite element theory is a new numerical analysis method, which is became the hot topics in the study of many scholars at home and abroad. Based on thin plate as the research object, this paper constructs the two-dimensional tensor product in the form of cubic Hermite scale function as unit interpolation function. For the partial differential equation and the total potential energy functional of the rectangular thin plate, thin plate element was constructed. And the bending and vibration problems of rectangular thin plate was studied by different boundary conditions. The numerical example shows that the result can be achieved a high analysis accuracy, so the effectiveness of the method was verified.

Key words. Wavelet finite element, thin plate, bending and vibration problem, hermite scaling function.

1. Introduction

In recent years, wavelet theory is introduced into the finite element method, the formation of the wavelet finite element theory is becoming a hot spot of research. The wavelet finite element theory as a new numerical analysis method, using the scale function and wavelet function as the interpolation function, has been used in numerical analysis, signal processing and engineering structure finite element analysis of machinery, such as the application research by the domestic and foreign scholars[1-8]. For the bending and vibration problem of thin plate, this paper constructs the two-dimensional tensor product in the form of cubic Hermite scale function. And in a variety of different boundary conditions, based on two-dimensional Hermite wavelet in solving maximum deflection and thin plate bending problem solving thin plate first 5 order natural frequency of free vibration, the theoretical solution of the approximation effect is remarkable, illustrates the effectiveness of the method.

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2. The structure of a two-dimensional Hermite wavelet unit

A one-dimensional wavelet is a function of single variable, on the basis of the use of tensor product, single variable function can be converted into bivariate function.In this paper, namely on the basis of one dimensional cubic Hermite function, and combining with the related knowledge of tensor product, using tensor product scaling function to construct the 2-d Hermite wavelet.Wavelet units as shown in the following type:

$$\Phi\left(\xi,\eta\right)=\Phi_1\left(\xi\right)\otimes\Phi_2\left(\eta\right) \tag{1}$$

Type of symbol Kronecker product, it is a special form of tensor product. $\Phi_1\left(\xi\right)$ and $\Phi_2\left(\eta\right)$ are one-dimensional cubic Hermite scale functions, expressed as

$$\Phi_1\left(\xi\right)=\left[H_1^{(0)}\left(\xi\right),H_1^{(1)}\left(\xi\right),H_2^{(0)}\left(\xi\right),H_2^{(1)}\left(\xi\right)\right] \tag{2}$$

$$\Phi_2\left(\eta\right)=\left[H_1^{(0)}\left(\eta\right),H_1^{(1)}\left(\eta\right),H_2^{(0)}\left(\eta\right),H_2^{(1)}\left(\eta\right)\right] \tag{3}$$

Type (2), (3) the expression of each function is as follows

$$\begin{array}{ll}
H_1^{(0)}\left(\xi\right)=1-3\xi^2+2\xi^3, & H_1^{(0)}\left(\eta\right)=1-3\eta^2+2\eta^3 \\
H_2^{(0)}\left(\xi\right)=3\xi^2-2\xi^3, & H_2^{(0)}\left(\eta\right)=3\eta^2-2\eta^3 \\
H_1^{(1)}\left(\xi\right)=\xi-2\xi^2+\xi^3, & H_1^{(1)}\left(\eta\right)=\eta-2\eta^2+\eta^3 \\
H_2^{(1)}\left(\xi\right)=\xi^3-\xi^2, & H_2^{(1)}\left(\eta\right)=\eta^3-\eta^2
\end{array}$$

The 2-d Hermite interpolation function of expression can be obtained by Equation (2), (3) into (1), and are selected as shown in figure 1 of the 2-d Hermite wavelet unit.

3. The structure of plate unit

As shown in figure 2, rectangular elastic thin plates, the solution domain is Ω; the length of unit is respectively $L_x$and L_y.

(1)In structural finite element analysis, the total potential energy functional expression of the thin plate bending is

$$\Pi_p=\frac{1}{2}\iint_\Omega \kappa^T D\kappa dxdy-\iint_\Omega wqdxdy \tag{4}$$

where Ω is the solution domain.$w$is the displacement field function, namely deflection. D is the elastic matrix of thin plate structure,

$$D=D_0\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & (1-\mu)2 \end{bmatrix} \tag{5}$$

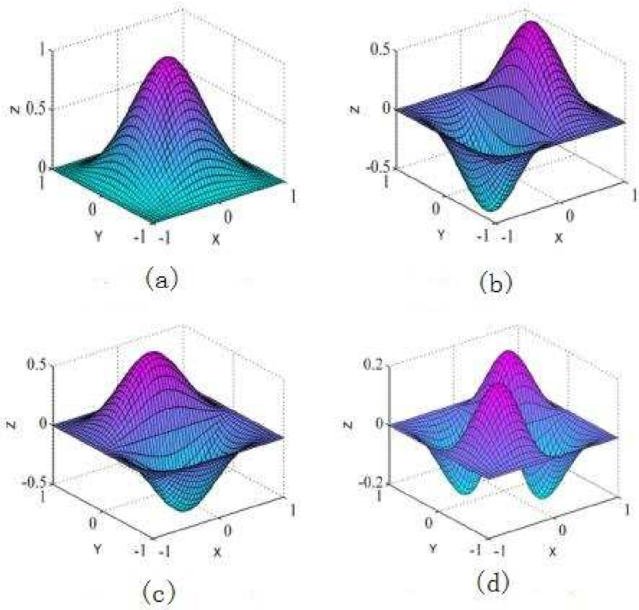

Fig. 1. The two-dimensional hermite wavelet unit

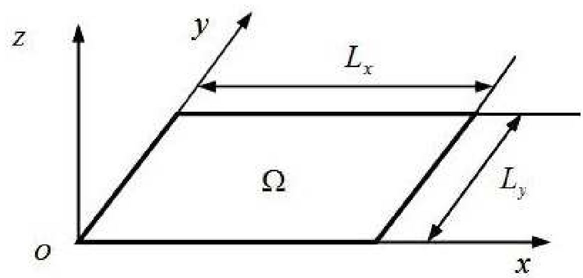

Fig. 2. Thin plate bending rectangular solution domain Ω

where D_0 is the bending stiffness, which means the elastic constants of plate, $D_0 = \frac{Et^3}{12(1-\mu^2)}$

where E is the elastic modulus, t is the plate thickness, $\mu$is the Poisson's ratio, κ is the generalized strain array, as showed in the following expression:

$$\kappa = \left[-\frac{\partial^2 w}{\partial x^2}, -\frac{\partial^2 w}{\partial y^2}, -2\frac{\partial^2 w}{\partial x \partial y} \right]^T \tag{6}$$

$(n-1) \times (n-1)$ grid, the arrangement of unit nodes as shown in figure 3, the length and width of the unit is respectivelyl_x,l_y.

Suppose that n=2^j+1(j is the space scales), the unit nodes are arranged in equal interval and are divided into$(n-1) \times (n-1)$ grid, the total number of unit nodes are

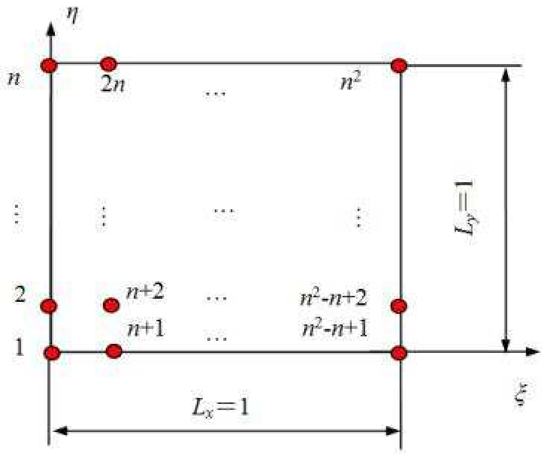

Fig. 3. Nodes are arranged on the solution domain units

$n \times n$, the freedom degree of each unit node is 4, So the total freedom degree of unit is $4 \times n \times n$. At this moment, the unknown displacement filed function can be expressed as

$$w{=}\Phi a \tag{7}$$

where, the wavelet coefficient is $a{=}\left[a_1, a_2, \cdots, a_{16}\right]^T$, the wavelet interpolation function $\Phi$is two-dimensional cubic Hermite wavelet, namely

$$\Phi = \Phi_1 \otimes \Phi_2 \tag{8}$$

Will type (7), (8) into type (6) can be obtained another kind of expression of the generalized matrix

$$\kappa = \begin{bmatrix} -\Phi''(\xi) \otimes \Phi(\eta) a \\ -\Phi(\xi) \otimes \Phi''(\eta) a \\ -2\Phi'(\xi) \otimes \Phi'(\eta) a \end{bmatrix} \tag{9}$$

Will equation (7),(9) into equation (4) and combine with the variational principle, $\delta\Pi_p = 0$, the finite element equations of the thin plate can be written as

$$\tilde{K}a = \tilde{P} \tag{10}$$

Where $\tilde{K}$ is the stiffness matrix of wavelet space,

$$\begin{aligned}\tilde{K}{=}D_0 \Big[&\Gamma_1^{2,2} \otimes \Gamma_2^{0,0}{+}\mu\Gamma_1^{0,2} \otimes \Gamma_2^{2,0}{+}\mu\Gamma_1^{2,0} \otimes \Gamma_2^{0,2} \\ &+ \Gamma_1^{0,0} \otimes \Gamma_2^{2,2}{+}2\left(1-\mu\right)\Gamma_1^{1,1} \otimes \Gamma_2^{1,1}\Big]\end{aligned} \tag{11}$$

$\tilde{P}$ is the load array of wavelet space,

$$\tilde{P}=l_x l_y \int_0^1 \int_0^1 q(\xi,\eta)\,\Phi^T(\xi)\,\Phi^T(\eta)\,d\xi d\eta \tag{12}$$

In addition, in the equation (11), each integral item is calculated respectively as follows

$$\Gamma_1^{2,2}=\frac{1}{l_x^3}\int_0^1 \left(\Phi''(\xi)\right)^T \Phi''(\xi)\,d\xi$$

$$\Gamma_1^{0,2}=\frac{1}{l_x}\int_0^1 \left(\Phi(\xi)\right)^T \Phi''(\xi)\,d\xi;\Gamma_1^{2,0}=\left(\Gamma_1^{0,2}\right)^T$$

$$\Gamma_1^{1,1}=\frac{1}{l_x}\int_0^1 \left(\Phi'(\xi)\right)^T \Phi'(\xi)\,d\xi;\Gamma_1^{0,0}=l_x\int_0^1 \left(\Phi(\xi)\right)^T \Phi(\xi)\,d\xi$$

using l_y,$d\eta$ respectively to replace l_x, $d\xi$in $\Gamma_1^{i,j}$ $(i,j{=}0,1,2)$, $\Gamma_2^{i,j}$ $(i,j{=}0,1,2)$ will be obtained.

(2)In structural finite element analysis, the total potential energy of rectangular thin plate in vibration is

$$\Pi_p=\frac{1}{2}\iint_\Omega \kappa^T D\kappa dxdy-\frac{1}{2}\iint_\Omega \rho t\omega^2 w^T w dxdy \tag{13}$$

where, using $\rho$is the density of material, $w$is the inherent frequency, t is the thickness.

with the principle of minimum potential energy, displacement field function is processed, $\delta\Pi_p=0$, the rectangular thin plate vibration frequency equation is

$$\left(\tilde{K}-\omega^2\tilde{M}\right)a=0 \tag{14}$$

where, $\tilde{K}$ is stiffness matrix, which is like equation (12); $\tilde{M}$ is the mass matrix, which could be obtained with the following equation $\tilde{M}=l_x l_y \rho t \Gamma_1^{0,0}\otimes\Gamma_2^{0,0}$

4. Numerical example

Example 1, there is a square thin plate which under uniform distributed load $q(x,y)=1000$ N/m^2, the length is $L{=}1$ m, the thickness is $t{=}0.1$ m, the elastic modulus is $E{=}3\times10^{11}$ Pa, the Poisson's ratio is $\mu{=}0.3$. Under different boundary conditions, try to solve the maximum deflection of the thin plate.

In the solving above problem, choosing wavelet unit as shown in figure 1. Based on the wavelet finite element theory and MATLAB software, the thin plate structure is divided into 256 units (16×16 mesh), a total of 289 nodes, 1156 degrees of freedom. Under the different boundary conditions, the wavelet finite element solution of the maximum deflection about thin plate and the corresponding theoretical solution are shown in table 1.

Based on the two-dimensional cubic Hermite wavelet, the maximum deflection of the thin plate with in a variety of different boundary conditions in the table 1, which is similar to the theoretical solution. The relative error between Hermite wavelet finite element solution and the theoretical solution is less than 1.

Example 2, there is a square sheet free vibration analysis with the length is$L$=1 m, the thickness is t=0.1 m the Poisson's ratio is μ=0.3, the density is ρ=7.9 × 10^3 kg/m^3, the elastic modulus is E=2 × 10^{11} Pa.

In the solving above problem, still choose wavelet unit as shown in figure 1, and the thin plate structure is divided into 256 units (16×16 mesh), a total of 289 nodes, 1156 degrees of freedom. Under the different boundary conditions, a square plate finite element solution of the first five order natural circle frequency and the corresponding theoretical solution are shown in table 2.

Based on the two-dimensional cubic Hermite wavelet, 5 order natural frequency of free vibration are obtained with in a variety of different boundary conditions in the table 2, which is similar to the theoretical solution. The relative error between Hermite wavelet finite element solution and the theoretical solution is less than 5‰.

Table 1 The maximum deflection of a square sheet and corresponding theoretical solution

Boundary conditions	Hermite wavelet finite element solution (10^{-7})	The theoretical solution (10^{-7})	The relative error (‰)
CCCC	0.46057	0.46053	0.087
SSSS	1.47870	1.47871	0.007
CSCS	0.69784	0.69888	1.49
CSSS	1.01874	1.01920	0.451

Table 2 Square plate first 5 order natural circle frequency finite element solutions and the corresponding theoretical solution

Boundary conditions	method	1 order (rad/s)	2 order (rad/s)	3 order (rad/s)	4 order (rad/s)	5 order (rad/s)
SSSS	Hermite wavelet finite element solution	19.8348	49.5020		79.3443	98.8758
	The theoretical solution	19.74	49.35		78.95	98.64
	The relative error /‰	4.985	3.080		4.994	2.391
CCCC	Hermite wavelet finite element solution	35.9856	73.3969		108.2231	131.5998
	The theoretical solution	35.99	73.41		108.3	131.6
	The relative error /‰	0.122	0.178		0.710	0.002
CCSS	Hermite wavelet finite element solution	29.0322	54.9098	69.4596	94.9656	102.4182
	The theoretical solution	28.95	54.74	69.32	94.59	102.2
	The relative error /‰	2.839	3.102	2.014	3.971	2.135
CSSS	Hermite wavelet finite element solution	23.7377	51.8368	58.7895	86.5207	100.4611
	The theoretical solution	23.64	51.67	58.65	86.12	100.30
	The relative error /‰	4.133	3.228	2.379	4.653	1.606

5. Conclusions

Based on in the form of two-dimensional tensor product cubic Hermite wavelet scale function as interpolation function, and the partial differential equation and the total potential energy functional of thin plate, the wavelet FEM thin plate unit. On the bending and vibration problems of thin plate related to wavelet finite element analysis, the corresponding numerical example showed that the Hermite wavelet finite element solution approximation to theoretical solution, and the feasibility of the method. So it provided a new research method.to study the related problem of more complex engineering structure.

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